We consider ~~the~~ a standard LPP

Maximize $z = cx$

Subject to $Ax = b$

and $x \geq 0$

$A = [a_{ij}]_{m \times n}$, $x = [x_1 \; \cdot \; x_n]^T$, $b = [b_1 \cdots b_n]^T$

Th: The set of all feasible solutions of a linear programming problem is a convex set.

Proof: We consider the LPP

$Ax = b$

together with the non negativity condition

where $A = [a_{ij}]_{m \times n}$, $x = [x_1 \; x_2 \ldots x_n]^T$

$b = [b_1 \; b_2 \ldots b_n]^T$.

Let X be the set of all feasible solutions.

Let $x_1, x_2 \in X$ Then $Ax_1 = b$, $Ax_2 = b$

together with $x_1, x_2 \geq 0$

Let $u = \lambda x_1 + (1-\lambda) x_2 \quad 0 \leq \lambda \leq 1$

be the convex combination of x_1, x_2

$Au = A(\lambda x_1) + A((1-\lambda)) x_2$

$= \lambda A x_1 + (1-\lambda) A x_2$

$= \lambda b + (1-\lambda) b = b$

and $u \geq 0$ $[\because$ ~~...~~ $\lambda x_1 \geq 0$ and $(1-\lambda) x_2 \geq 0]$

So ~~...~~ x_3 is a feasible solution.

Hence X is convex.

Cor:

If an LPP has two feasible solution then it has infinite no of feasible solutions.

Definition:

Suppose we have the LPP

Optimize $z = \sum_{j=1}^{n} c_j x_j$

subject to $\sum_{j=1}^{n} a_{ij} x_j \; (\leq = \geq) \; b_i \quad i = 1, \cdots, m.$

and $x_j \geq 0 \quad j = 1, \cdots n.$

~~A set~~ Feasible solution: A set of values of the ~~Feas~~ decision variables $x_1, x_2, \cdots, x_n$ which satisfies all the constraints including non-negativity restricts. is known as feasible solution.

~~Objective~~

Optimal solution: A feasible solution which optimizes the objective function is called the optimal solution.

Basic Solution: ~~Can We~~ .

We consider a system of equations in n variables

$Ax = b$

Suppose rank of $A = m$

So we can choose a $m \times m$ matrix from A and rest $n-m$ ~~as~~ variables are set 0. Then the solution of the system is known as basic feasible solution.